

Efficient & Reliable Hybrid DFT Calculations with ADMM

Sanliang Ling and Ben Slater

Email: S.Ling@ucl.ac.uk

Department of Chemistry
University College London

Second Annual CP2K-UK Users Meeting, London, 6th February 2015

Hybrid DFT Calculations with CP2K

- ADMM: Auxiliary Density Matrix Methods for Hartree-Fock Exchange Calculations
- Total energy as a functional of the electron density

$$E[\rho] = T_s[\rho] + J[\rho] + E_{xc}[\rho] + \int v(\mathbf{r}) \rho(\mathbf{r}) d\mathbf{r}$$

- Exchange-correlation energy with a hybrid functional

$$E_{xc}[\rho] = \alpha E_x^{\text{HFX}}[\{\psi_i\}] + (1 - \alpha) E_x^{\text{DFT}}[\rho] + E_c^{\text{DFT}}[\rho]$$

ADMM in CP2K

- Hartree-Fock exchange energy

$$E_x^{\text{HFX}}[P] = -\frac{1}{2} \sum_{\lambda\sigma\mu\nu} P^{\mu\sigma} P^{\nu\lambda} (\mu\nu|\lambda\sigma) \longrightarrow \text{scales as } N^4$$

$$P^{\mu\nu} = \sum_i C^{ui} C^{vi} \Leftrightarrow P = CC^T$$

$$(\mu\nu|\lambda\sigma) = \int \int \phi_\mu(\mathbf{r}_1) \phi_\nu(\mathbf{r}_1) g(|\mathbf{r}_2 - \mathbf{r}_1|) \phi_\lambda(\mathbf{r}_2) \phi_\sigma(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2$$

- Introducing auxiliary density matrix $\hat{P} \approx P$

$$\begin{aligned} E_x^{\text{HFX}}[P] &= E_x^{\text{HFX}}[\hat{P}] + (E_x^{\text{HFX}}[P] - E_x^{\text{HFX}}[\hat{P}]) \\ &\approx E_x^{\text{HFX}}[\hat{P}] + (E_x^{\text{DFT}}[P] - E_x^{\text{DFT}}[\hat{P}]) \end{aligned}$$

- How to construct auxiliary basis set?

- smaller in size (i.e. less number of basis functions)
- more rapidly decaying (i.e. bigger Gaussian exponents)

ADMM in CP2K

Choice of auxiliary basis set for ADMM

- FIT3: three Gaussian exponents for each valence orbital
- cFIT3: a contraction of FIT3 (i.e. fixed linear combinations of Gaussian functions)
- pFIT3: FIT3 + polarization functions (i.e. higher angular momentum functions)  **exponents taken from 6-31G** (unoptimised)**
- cpFIT3: cFIT3 + polarization functions
- aug-FIT3, aug-cFIT3, aug-pFIT3, aug-cpFIT3: augmented with a “diffuse” function (i.e. smaller Gaussian exponents)
- **FIT3 as trial ADMM basis**

ADMM in CP2K

1A																8A	
1 H	2A															2 He	1s ²
3 Li	4 Be	1s ² 2s ¹	1s ² 2s ²														
11 Na	12 Mg	[Ne]3s ¹	[Ne]3s ²														
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
[Ar]4s ¹	[Ar]4s ²	[Ar]3d ¹ 4s ²	[Ar]3d ² 4s ²	[Ar]3d ³ 4s ²	[Ar]3d ⁴ 4s ¹	[Ar]3d ⁵ 4s ²	[Ar]3d ⁶ 4s ²	[Ar]3d ⁷ 4s ²	[Ar]3d ⁸ 4s ²	[Ar]3d ⁹ 4s ¹	[Ar]3d ¹⁰ 4s ²						
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
[Kr]5s ¹	[Kr]5s ²	[Kr]4d ¹ 5s ²	[Kr]4d ² 5s ²	[Kr]4d ⁴ 5s ¹	[Kr]4d ⁵ 5s ¹	[Kr]4d ⁵ 5s ²	[Kr]4d ⁷ 5s ¹	[Kr]4d ⁸ 5s ¹	[Kr]4d ¹⁰	[Kr]4d ¹⁰ 5s ¹	[Kr]4d ¹⁰ 5s ²						
55 Cs	56 Ba	57-71 Lanthanides	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
[Xe]6s ¹	[Xe]6s ²	Lanthanides	[Xe]4f ¹⁴ 5d ² 6s ²	[Xe]4f ¹⁴ 5d ³ 6s ²	[Xe]4f ¹⁴ 5d ⁶ 6s ¹	[Xe]4f ¹⁴ 5d ¹⁰ 6s ¹	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²				
87 Fr	88 Ra	89-103 Actinides	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
[Rn]7s ¹	[Rn]7s ²	Actinides	[Rn]5f ¹⁴ 6d ² 7s ² *	[Rn]5f ¹⁴ 6d ³ 7s ² *	[Rn]5f ¹⁴ 6d ⁴ 7s ² *	[Rn]5f ¹⁴ 6d ⁵ 7s ² *	[Rn]5f ¹⁴ 6d ⁶ 7s ² *	[Rn]5f ¹⁴ 6d ⁷ 7s ² *	[Rn]5f ¹⁴ 6d ⁸ 7s ¹ *	[Rn]5f ¹⁴ 6d ⁹ 7s ¹ *	[Rn]5f ¹⁴ 6d ¹⁰ 7s ¹	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²					

Limited availability of ADMM basis sets

(see \$CP2K/cp2k/data/BASIS_ADMM)

ADMM basis sets for transition metals

1A	8A																	
1 H	2A	2 He																
3 Li	4 Be																	
11 Na	12 Mg																	
[Ne]3s ¹	[Ne]3s ²	3B	4B	5B	6B	7B	8B		1B	2B								
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
[Ar]4s ¹	[Ar]4s ²	[Ar]3d ¹ 4s ²	[Ar]3d ² 4s ²	[Ar]3d ³ 4s ²	[Ar]3d ⁴ 4s ¹	[Ar]3d ⁵ 4s ²	[Ar]3d ⁶ 4s ²	[Ar]3d ⁷ 4s ²	[Ar]3d ⁸ 4s ¹	[Ar]3d ⁹ 4s ¹	[Ar]3d ¹⁰ 4s ²	[Ar]3d ¹⁰ 4s ²						
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
[Kr]5s ¹	[Kr]5s ²	[Kr]4d ¹ 5s ²	[Kr]4d ² 5s ²	[Kr]4d ⁴ 5s ¹	[Kr]4d ⁵ 5s ¹	[Kr]4d ⁷ 5s ¹	[Kr]4d ⁸ 5s ¹	[Kr]4d ¹⁰	[Kr]4d ¹⁰ 5s ¹	[Kr]4d ¹⁰ 5s ²	[Kr]4d ¹⁰ 5s ²							
55 Cs	56 Ba	57-71 Lanthanides	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	
[Xe]6s ¹	[Xe]6s ²	Lanthanides	[Xe]4f ¹⁴ 5d ² 6s ²	[Xe]4f ¹⁴ 5d ³ 6s ²	[Xe]4f ¹⁴ 5d ⁵ 6s ²	[Xe]4f ¹⁴ 5d ⁶ 6s ²	[Xe]4f ¹⁴ 5d ⁸ 6s ²	[Xe]4f ¹⁴ 5d ⁹ 6s ¹	[Xe]4f ¹⁴ 5d ¹⁰ 6s ¹	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²								
87 Fr	88 Ra	89-103 Actinides	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo	
[Rn]7s ¹	[Rn]7s ²	Actinides	[Rn]5f ¹⁴ 6d ² 7s ² *	[Rn]5f ¹⁴ 6d ⁴ 7s ² *	[Rn]5f ¹⁴ 6d ⁷ 7s ² *	[Rn]5f ¹⁴ 6d ⁹ 7s ² *	[Rn]5f ¹⁴ 6d ⁷ 7s ¹ *	[Rn]5f ¹⁴ 6d ⁹ 7s ¹ *	[Rn]5f ¹⁴ 6d ¹⁰ 7s ¹	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²								

ADMM basis sets will be released in mid-2015!
 (Email: S.Ling@ucl.ac.uk)

ADMM basis sets for transition metals

Uncontracted basis sets

- FIT10: $4s + 3p + 3d$
- FIT11: $4s + 3p + 3d + 1d$ → recommended for solids
- FIT12: $4s + 3p + 4d + 1d$
- FIT13: $4s + 4p + 4d + 1d$

Contracted basis sets (double- ζ quality)

- cFIT10 / cFIT11 / cFIT12 / cFIT13

All exponents were optimised, including the polarisation function

Some general suggestions

- Always check the convergence of **CUTOFF**
(see http://www.cp2k.org/howto:converging_cutoff)
- Always check the convergence of properties (e.g. lattice parameters, band gaps) with respect to **supercell sizes**
- Always start from pre-converged GGA (e.g. PBE) wavefunction and geometry
- Always check the convergence of **primary and ADMM basis sets** (start from a small basis and gradually increase the size)
- ADMM has only been implemented for use with GPW

Work Flow

- 1. Convergence test (primary basis, CUTOFF, supercell, etc)**
 - 2. GGA optimisation with selected primary basis**
 - 3. Name GGA wave function file for use with ADMM**
 - 4. Construct ADMM input with auxiliary basis (e.g. FIT3)**
 - 5. Run calculations and check convergence of ADMM basis**
- 
- A vertical sequence of five green downward-pointing arrows, each positioned between two consecutive numbered steps to indicate a sequential process.

Input Structure: GGA/PBE

&DFT

...
BASIS_SET_FILE_NAME ./BASIS_MOLOPT (file can be found in \$CP2K/cp2k/data)

...

&XC

&XC_FUNCTIONAL PBE
&END XC_FUNCTIONAL

&END XC

...

&END DFT

&SUBSYS

&KIND Si

BASIS_SET DZVP-MOLOPT-SR-GTH
POTENTIAL GTH-PBE-q4

&END KIND

&END SUBSYS

Input Structure: ADMM

&DFT

```
...
BASIS_SET_FILE_NAME ./BASIS_MOLOPT      (files can be found in $CP2K/cp2k/data)
BASIS_SET_FILE_NAME ./BASIS_ADMM
WFN_RESTART_FILE_NAME ${project}-RESTART.wfn
&SCF
    SCF_GUESS RESTART
...
&END SCF
&AUXILIARY_DENSITY_MATRIX_METHOD
    METHOD BASIS_PROJECTION
    ADMM_PURIFICATION_METHOD MO_DIAG
&END AUXILIARY_DENSITY_MATRIX_METHOD
...
&XC
...
&END XC
&END DFT
&SUBSYS
    &KIND Si
        BASIS_SET DZVP-MOLOPT-SR-GTH
        AUX_FIT_BASIS_SET cFIT3
        POTENTIAL GTH-PBE-q4
    &END KIND
&END SUBSYS
```

Which functional to use?

- PBE0-TC-LRC

$$E_{xc}^{PBE0-TC-LRC} = aE_x^{HF,TC}(R_C) + aE_x^{PBE,LRC}(R_C) \\ + (1 - a)E_x^{PBE} + E_c^{PBE}$$

J. Chem. Theory Comput., 5, 3010 (2009)

- HSE06

$$E_{xc}^{HSE06} = aE_x^{HF,SR}(\omega) + (1 - a)E_x^{PBE,SR}(\omega) \\ + E_x^{PBE,LR}(\omega) + E_c^{PBE}$$

J. Chem. Phys., 125, 224106 (2006)

Input Structure: PBE0 vs. HSE06

&XC

```

  &XC_FUNCTIONAL
  &PBE
    SCALE_X 0.75
    SCALE_C 1.0
  &END PBE
  &PBE_HOLE_T_C_LR
    CUTOFF_RADIUS 2.0
    SCALE_X 0.25
  &END PBE_HOLE_T_C_LR
  &END XC_FUNCTIONAL
  &HF
    &SCREENING
      EPS_SCHWARZ 1.0E-6
      SCREEN_ON_INITIAL_P FALSE
    &END SCREENING
    &INTERACTION_POTENTIAL
      POTENTIAL_TYPE TRUNCATED
      CUTOFF_RADIUS 2.0
      T_C_G_DATA ./t_c_g.dat
    &END INTERACTION_POTENTIAL
    &MEMORY
      MAX_MEMORY 2400
      EPS_STORAGE_SCALING 0.1
    &END MEMORY
    FRACTION 0.25
  &END HF
  &END XC

```

PBE0-TC-LRC

&XC

```

  &XC_FUNCTIONAL
  &PBE
    SCALE_X 0.0
    SCALE_C 1.0
  &END PBE
  &XWPBE
    SCALE_X -0.25
    SCALE_X0 1.0
    OMEGA 0.11
  &END XWPBE
  &END XC_FUNCTIONAL
  &HF
    &SCREENING
      EPS_SCHWARZ 1.0E-6
      SCREEN_ON_INITIAL_P FALSE
    &END SCREENING
    &INTERACTION_POTENTIAL
      POTENTIAL_TYPE SHORTRANGE
      OMEGA 0.11
    &END INTERACTION_POTENTIAL
    &MEMORY
      MAX_MEMORY 2400
      EPS_STORAGE_SCALING 0.1
    &END MEMORY
    FRACTION 0.25
  &END HF
  &END XC

```

HSE06

(see examples in \$CP2K/cp2k/tests/QS/regtest-admm-1/2/3/4)

Example: Diamond Band Gap

method	number of integrals	gap [eV]
PBE (PBS)		4.17
PBE (ABS)		4.37
PBE0 (PBS)	40 787 850 778 591	6.07
PBE0 (ABS)	23 561 509 497	6.25
PBE0 ADMM1	24 816 897 009	6.03
PBE0 ADMM2	24 795 460 638	6.02

3x3x3 supercell

Example: Bulk Silicon

Cutoff radius (\AA)	Band gap (eV)	Integrals
2	1.16 ^a	77799946176
4	1.54 ^a	154325979000
6	1.71 ^a	265868148312
8	1.78 ^a	422457823080

PBE0-TC-LRC with cFIT3 ADMM basis, 3x3x3 supercell, 216 atoms

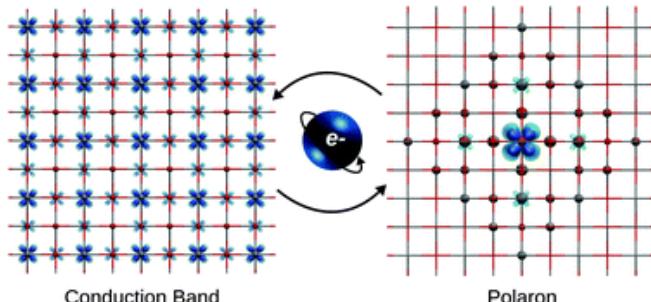
ADMM basis	Band gap (eV)	Integrals
cFIT3	1.78 ^a	422457823080
FIT3	1.80 ^a	424426850352
pFIT3	1.98 ^a	1447428361680
Ref. (VASP)	1.93 ^b (indirect)	

PBE0-TC-LRC with 8 \AA cutoff radius, 3x3x3 supercell, 216 atoms

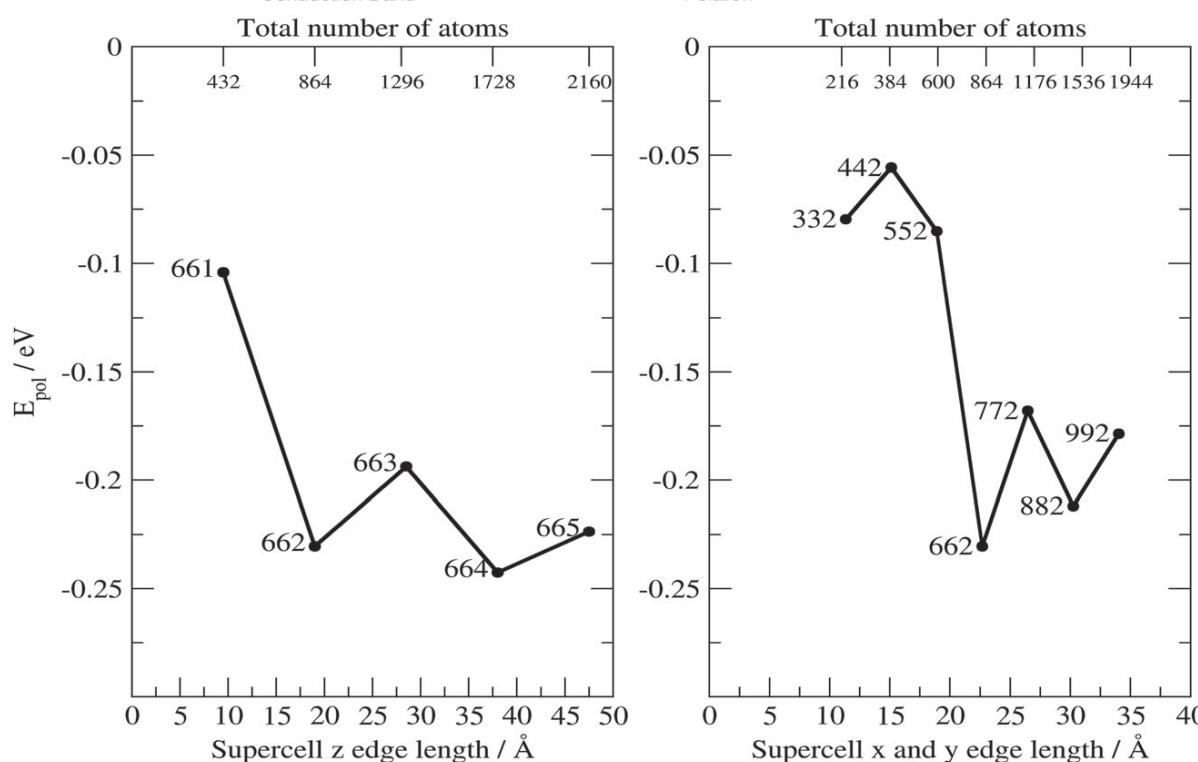
Cutoff radius
 $R_C \leq \frac{L}{2}$

Polarisation
function is
important for
covalent solids!

Example: excess electrons in TiO_2

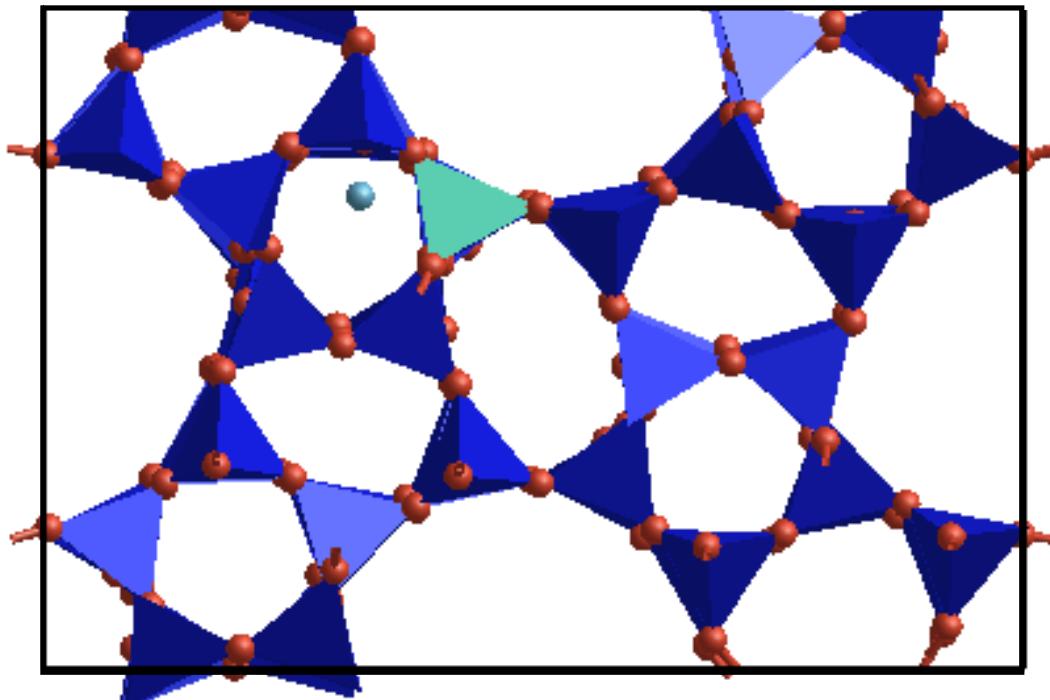


$$E_{\text{pol}} = E_{\text{loc}(N+1)} - E_{\text{deloc}(N+1)}$$



at 298 K,
 $kT = \sim 0.03 \text{ eV}$

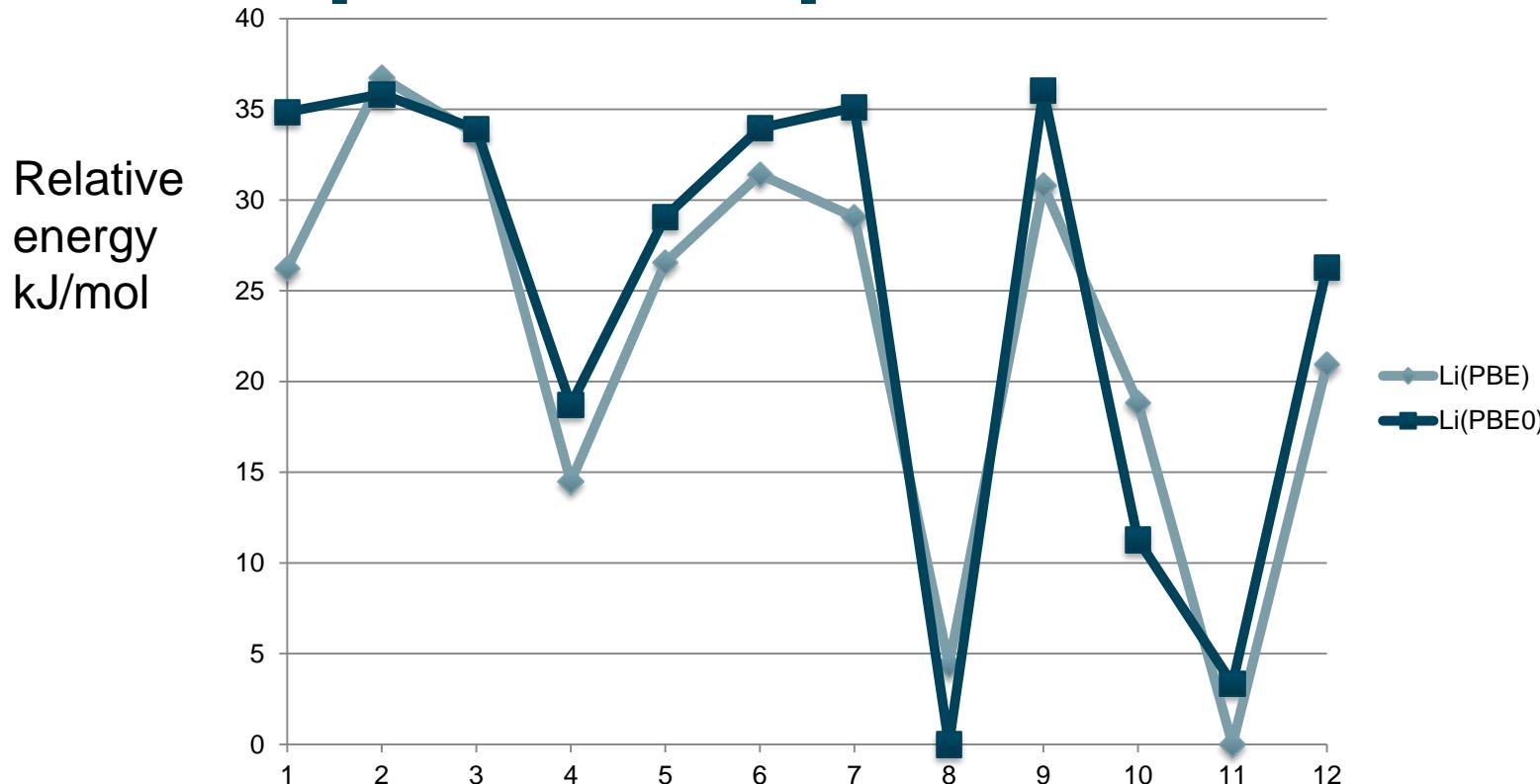
Example: Li doped ZSM-5



- Widely used nanoporous zeolite catalyst
- $\text{Si}_{1-x}\text{O}_2\text{Al}_x\cdot\text{Li}_x$
- 289 atoms in the unit cell
- 95 Si (dark blue), 1 Al (green), Li (light blue)
- 12 symmetry distinct positions for the Al and 48 distinct positions for the Li cation
- $2 \times 2 \times 1.3\text{nm}$ cell
- TZV2P primary basis/ PBE

- Open question in zeolite science – what determines where the Al is found in this material?
- Important, because Al is associated with acidity and catalytic activity

Example: Li doped ZSM-5



- Preference for two lattice sites at PBE and hybrid PBE0 level of theory
- 1 energy evaluation with full hybrid ~2 hours on 480 cores (ARCHER)
- ADMM (CFIT), full geometry optimisation in 6 hours on 480 cores (ARCHER) – 1936 energy evaluations, 293 optimisation steps!

Acknowledgements

Prof Michiel Sprik

Dr Matt Watkins

Dr Florian Schiffmann

Funding



Pioneering research
and skills

Computing



UK HPC Materials Chemistry Consortium